Explicit Derivation of M2-Brane Charge Quantization in M-Theory

Douglas G. Stevenson
stevensonfluxinformationtheory.com

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1 Introduction

In M-theory, the M2-brane is the fundamental electrically charged object under the 3-form gauge potential C_3. Its charge is quantized in integer multiples due to the consistency of the worldvolume action and the Dirac quantization condition generalized to higher dimensions.

This document derives the M2-brane charge quantization condition step by step, including the explicit large gauge transformation calculation and a diagram of the linking S^7.

2 M2-Brane Worldvolume Action

The low-energy effective action for a single M2-brane is

$$S_{\mathrm{M2}} = -T_2 \int d^3\xi \sqrt{-\det(\gamma_{ij})} + T_2 \int_{\text{worldvolume}} C_3,$$

where $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ is the M2-brane tension and γ_{ij} is the induced metric.

3 Field Strength and Flux

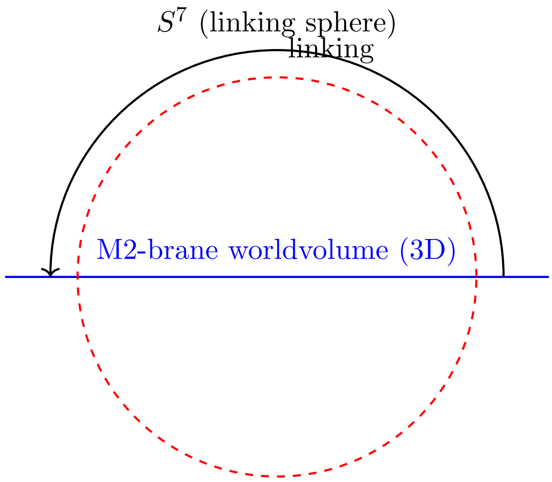
The 4-form field strength is $F_4 = dC_3$. The electric charge of the M2-brane is

$$Q_2 = \int_{S^7} *F_4,$$

where the integral is over a 7-sphere linking the M2-brane worldvolume.

4 Diagram: S^7 Linking the M2-Brane

The M2-brane worldvolume is a 3-dimensional surface in 11D spacetime. A 7-sphere S^7 that links this worldvolume measures the enclosed flux.



Schematic in 11D: M2 worldvolume pierced by linking S^7

Figure 1: Schematic illustration of an S^7 linking the M2-brane worldvolume. The integral $\int_{S^7} *F_4$ measures the enclosed M2-brane charge.

5 Explicit Large Gauge Transformation Calculation

Consider a large gauge transformation of the 3-form:

$$C_3 \to C_3 + d\Lambda_2,$$

where Λ_2 is a 2-form with integer winding number around a 3-cycle.

The change in the Wess-Zumino term is

$$\Delta S_{\mathrm{WZ}} = T_2 \int_{\text{worldvolume}} d\Lambda_2 = T_2 \int_{\partial\Sigma_3} \Lambda_2,$$

where Σ_3 is a 3-chain whose boundary is the worldvolume.

Using Stokes' theorem and the linking S^7, this becomes

$$\Delta S_{\mathrm{WZ}} = T_2 \int_{S^7} *F_4.$$

For the quantum theory to be consistent, the path-integral phase factor must be single-valued:

$$e^{i\Delta S_{\mathrm{WZ}}} = e^{iT_2 \int_{S^7} *F_4} = 1 \quad (\text{or } e^{2\pi i n} \text{ for integer } n).$$

Substituting the tension $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ yields

$$T_2 \int_{S^7} *F_4 = 2\pi n, \quad n \in \mathbb{Z}.$$

Thus,

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3.$$

This is the M2-brane charge quantization condition.

6 Magnetic Dual: M5-Brane Quantization

The magnetic dual is

$$\int_{S^4} F_4 = 2\pi m\, \ell_{11}^6, \quad m \in \mathbb{Z}.$$

7 Connection to SFIT

M-theory quantizes M2-brane flux at the Planck scale. SFIT describes an effective low-energy resonant information flux at $\nu_{\text{res}} = 1.20134\,\text{mHz}$ with coupling kernel $K = 1.060$.

The quantized M2-brane flux may be the microscopic origin of the SFIT information-carrying flux. When M2-branes interact with a macroscopic gravitational field, they produce the observed 1.20134 mHz modulation and KWW tails with $\beta = K = 1.060$.

The non-reciprocal metric correction in SFIT could be the coarse-grained signature of quantized M2-brane flux back-reaction.

8 Conclusion

The M2-brane charge quantization condition is

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3, \quad n \in \mathbb{Z}.$$

This follows from requiring single-valuedness of the worldvolume path integral under large gauge transformations of C_3. The explicit calculation shows $\Delta S_{\text{WZ}} = 2\pi n$.

SFIT may capture the effective resonant behavior of these quantized fluxes at laboratory energies.